TAYLOR POLYNOMIAL

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**Abstract**

The Taylor series stands as a fundamental and versatile concept in mathematics, providing a robust framework for approximating functions through infinite sums of derivatives. Originating from the seminal work of Brook Taylor in 1715 and subsequently expanded by Colin Maclaurin, this series allows the expression of a function as an infinite sum of terms based on its derivatives at a single point. The resulting Taylor polynomials, formed by partial sums, systematically refine approximations as the degree increases, offering a powerful tool for local function approximation.

Crucial to the understanding and application of Taylor series is Taylor's theorem, which quantifies the error incurred by these approximations. This theorem plays a pivotal role in elucidating concepts of convergence and analyticity, providing mathematical rigor to the use of Taylor series in various domains.

In the contemporary mathematical landscape, the advent of computer algebra systems has further amplified the significance of Taylor series. These systems facilitate intricate computations, enabling swift and accurate calculations of Taylor series for a wide array of functions. exploring the structural intricacies, diverse applications, and historical evolution of Taylor series, showcasing its enduring importance in the realm of mathematical analysis. From its roots in 18th-century mathematical innovations to its modern computational applications, the Taylor series continues to be a cornerstone for understanding and approximating functions in diverse mathematical contexts.

**Introduction**

In the vast landscape of mathematical analysis, the Taylor series emerges as a pivotal concept, reshaping our approach to function approximation. First introduced by Brook Taylor in 1715 and later refined by Colin Maclaurin, the Taylor series stands as a versatile tool, enabling the representation of a function as an infinite sum based on its derivatives at a single point. This series, expressed by the formula:

Pn(x)=f(x0)+f′(x0)(x−x0)+ +…+

profoundly influences how we understand and work with functions in the vicinity of a chosen point.

The essence of the Taylor series lies in its ability to construct increasingly accurate approximations through Taylor polynomials, formed by partial sums of the series. These polynomials become indispensable tools for local function estimation, enabling a meticulous exploration of a function's behavior near a specific point.

Beyond its practical applications, the mathematical rigor of the Taylor series is underpinned by Taylor's theorem, providing a quantitative measure of approximation errors. This theorem is pivotal in unraveling the concepts of convergence and analyticity, ensuring a robust foundation for the utilization of Taylor series in diverse mathematical contexts.

**History**

The historical tapestry of the Taylor series unfolds across centuries, weaving together the contributions of mathematical luminaries who shaped its evolution. The series finds its origins in the ancient musings of Greek philosophers like Zeno of Elea, who grappled with the concept of infinite summation. Archimedes, in the third century BCE, introduced the method of exhaustion, laying the groundwork for later developments.

In the 17th century, James Gregory, unaware of Isaac Newton's concurrent efforts, explored series expansions, setting the stage for significant discoveries. Newton, in 1691–1692, explicitly wrote down the Taylor and Maclaurin series in his unpublished work, laying the theoretical foundation. However, it was Brook Taylor who, in 1715, offered a general method for constructing these series for all functions, securing his place as a key figure in the series' history.

Colin Maclaurin's specialized treatment of the Taylor series at the point where derivatives are considered at 0, known as the Maclaurin series, further enriched its applicability. The series, once cumbersome in its early development, transformed into a foundational tool in mathematical analysis, influencing diverse fields and standing testament to the enduring legacy of mathematical innovation.

**Taylor Polynomial**

Formula:

The n-th order Taylor Polynomial at the x0 [a,b] point of the function f has the following form

(1)

Pn(x)=f(x0)+f′(x0)(x−x0)+ +…+ =

Where f C(n+1)[a,b]. The derivative of the Taylor Polynomial of n-th order including, as it appears from formula (1), at point x0 coincide with the corresponding value of the derivatives of the function f, i.e.

(x) = f(k)(x0), k=0,1,…,n.

A special case of the Taylor polynomial is the Maclaurin polynomial, where *c*=0. That is, the **Maclaurin polynomial of degree** n of f is

Pn(x)=f(0)+ f′(0)x + + + … +

A Maclaurin polynomial is a specific case of a Taylor polynomial where the center (c) is set to 0. Therefore, a Maclaurin polynomial is a Taylor polynomial with c=0.

**Example of where we can use Tayler Polynomial**

**Example 1.**

Calculate the fourth degree Taylor polynomial for the function f(x)=(x) at x = 0 at and use it to approximate .

Remember that a Taylor polynomial at x = 0 is called a Maclaurin polynomial! First, calculate the first 4 derivatives of f(x) = (x) and evaluate them at x = 0.

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Then, the fourth degree Taylor polynomial around is x = 0 is

**A math equations with numbers and symbols

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So, evaluating at x = 2 , you have

**A math equations with numbers and symbols

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**Example 2.**

Calculate the value of using a quadratic approximation.

**Solution:**

In this case, we need to calculate the second-degree Taylor polynomial of the function g(x)= since you want a quadratic approximate of .

Since Taylor polynomials only allow you to approximate values close to the value at which they are centered, you need a value close to 24 where you can actually find the square root easily. So let's take 25 since = 5.

A math equation with numbers and lines

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Thus, the quadratic (another way of saying second degree) Taylor polynomial of centered at x = 25 is

A math equation with numbers

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Using T2(x) the approximation you get

A number with numbers on it

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Taylor polynomials are used to approximate complex functions and allow you to calculate values that are difficult to compute.

**Python code to calculate Taylor Polynomial**

A screenshot of a computer program

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This Python code implements the Taylor series expansion of a user-entered mathematical function using the SymPy library. The user is prompted to input a function, the number of terms (steps) for the expansion, and a value for c or x0​. The code then calculates the derivatives of the function up to the specified order, and iterates through each derivative to compute the corresponding term in the Taylor series. The terms are added to the overall sum, and the code prints each term along with its order. Additionally, if a term is the same with and without brackets (assuming this refers to parentheses), it prints only one version; otherwise, it prints both. The accuracy of the Taylor series expansion depends on the function and the chosen expansion point. The code utilizes SymPy for symbolic computation and the math module for factorial calculations.

**Conclusion**

In conclusion, Taylor polynomials and series constitute indispensable tools in the realm of mathematical analysis, providing a robust framework for the approximation of complex functions. Rooted in a historical tapestry that extends from ancient Greek philosophy to the contributions of mathematicians such as Brook Taylor and Colin Maclaurin, these concepts have continually evolved to become foundational elements in contemporary mathematics. The power of Taylor polynomials lies in their ability to approximate functions with increasing accuracy as more terms are included, offering a valuable means of computation for values that may be challenging to obtain directly.

Taylor's theorem, underpinning the mathematical rigor of these polynomials, further enhances their utility, providing a systematic approach to understanding and estimating functions. Whether viewed through the lens of historical innovation or applied in modern computational contexts, the Taylor series stands as a cornerstone in the exploration of mathematical landscapes. Its adaptability, precision, and versatility make it an enduring and invaluable tool for mathematicians, scientists, and researchers alike, underscoring its significance in the ever-expanding pursuit of knowledge and understanding in the field of mathematics.